# Algorithmic Issues and Applications of BOUT to Simulation of Realistic Tokamak Configurations

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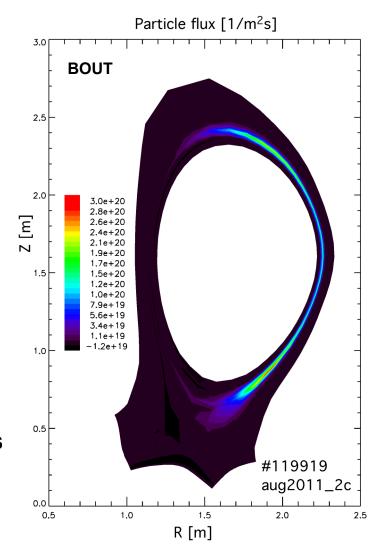


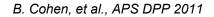
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# BOUT Simulations of Resistive Ballooning Turbulence in Edge Region for DIII-D Shot #119919

- Simulations of electrostatic resistive ballooning in DIII-D shot #119919, with full geometry and magnetic shear, crossing the separatrix
- Nonlinear BOUT equations for ion density, vorticity, electron and ion velocities, Ohm's law, and Maxwell's equations.
- In earlier work, we have suppressed a spatial odd-even mode ballooning along the field line by either filtering with  $\nabla_{||}^2$  or  $-\nabla_{||}^4$  diffusive operator added to right side of vorticity and ion density eqns, or with use of a staggered mesh for  $\nabla_{||}$  representation. Parallel damping included here; no odd-even mode seen.
- Simulation results with/without T<sub>e</sub> fluctuations
- BOUT obtains steady-state turbulence with fluctuation amplitudes and transport that compare reasonably to the DIII-D data.







### BOUT Simulation of Resistive Ballooning Turbulence for DIII-D Shot #119919 - Outline

- BOUT algorithmic issues -- control of an odd-even numerical contamination
- Electromagnetic simulations of resistive ballooning turbulence in single-null DIII-D geometry:
  - Case #1: No T<sub>e</sub> fluctuations
  - Case #2: With T<sub>e</sub> fluctuations
  - Case #3: With T<sub>e</sub> fluctuations and electron parallel thermal conduction
  - Case #4: With  $T_e$  fluctuations, electron parallel thermal conduction, and  $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$  in the vorticity eqn.
- Comparison to probe data for DIII-D shot #119919. Shot #119919 is a well-characterized L-mode shot exhibiting steady-state turbulence.



## BOUT06 produces expected ballooning-like turbulence in full DIII-D X-point geometry

 BOUT solves Braginskii-like fluid equations for fluid turbulence in various geometries

Distribution of <δN>

$$\frac{\partial N_{i}}{\partial t} + (V_{E} + V_{\parallel}) \bullet \nabla N_{i} = \left(\frac{2c}{eB}\right) b_{0} \times \kappa \bullet (\nabla P_{e} - N_{i}e\nabla \phi) + \nabla_{\parallel}(j_{\parallel}/e) - N_{i}\nabla_{\parallel}V_{\parallel i}$$

$$\frac{\partial \varpi}{\partial t} + V_E \bullet \nabla \varpi = 2\omega_{ci}b_0 \times \kappa \bullet \nabla P + N_i Z_i e^{\frac{4\pi V_A^2}{c^2}} \nabla_{\parallel} j_{\parallel} \text{ vorticity}$$

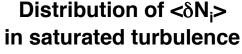
$$\frac{\partial V_{\parallel e}}{\partial t} = -\frac{e}{m_e} E_{\parallel} - \frac{1}{Nm_e} (T_e \nabla_{\parallel} N_i) + 0.51 v_{ei} j_{\parallel}$$

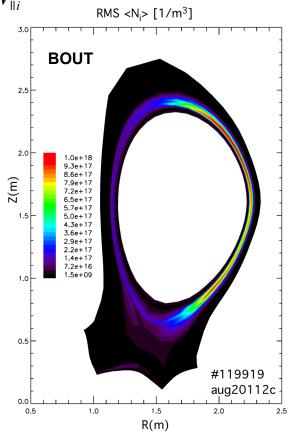
$$\frac{\partial V_{\parallel i}}{\partial t} + V_E \bullet \nabla V_{\parallel i} = -\frac{1}{N_i M_i} \nabla_{\parallel} P$$

$$\mathbf{E} = -\frac{1}{c} \frac{\partial}{\partial t} \mathbf{A}_{\parallel} - \nabla \phi, \quad -\nabla_{\perp}^{2} \mathbf{A}_{\parallel} = \frac{4\pi}{c} \mathbf{j}_{\parallel}, \quad \mathbf{B} = \nabla \times \mathbf{A}_{\parallel} + \mathbf{B}_{0}$$

$$\boldsymbol{\varpi} = \nabla \cdot (eZ_i N_i \nabla \phi) \approx eZ_i N_i \nabla^2 \phi \quad \nabla_{\parallel} \approx \mathbf{b}_0 \cdot \nabla$$

- Electromagnetic with  $\nabla_{\parallel} \approx \mathbf{b}_0 \cdot \nabla$
- Finite-difference equations
- Implicit time integration with PVODE
- Quasi-ballooning with zero-gradient radial bdry conditions





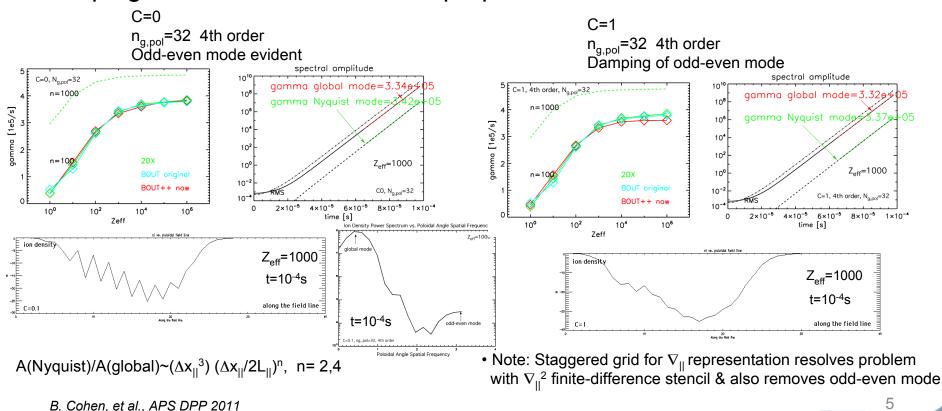


### Resistive Ballooning Simulations with BOUT -- Odd-even Numerical Mode Can Be Controlled with Normalized Diffusive Damping in Poloidal Angle

• Control the odd-even mode with  $\partial/\partial t \rightarrow \partial/\partial t + v^*(k_{\theta})$  in the vorticity and ion density eqns with diffusion operator in poloidal angle and normalized coeff.:

$$v^*(k_{\theta}) = \frac{C}{\Delta t} \left\{ -\left(\Delta\theta/2\right)^2 D_{\theta}^2, \left(\Delta\theta/2\right)^4 D_{\theta}^4 \right\} \rightarrow \frac{C}{\Delta t} \left\{ \sin\left(k_{\theta}\Delta\theta/2\right)^2, \sin\left(k_{\theta}\Delta\theta/2\right)^4 \right\}$$

• Damping of the odd-even mode is proportional to normalized coefficient C



## Case #1: BOUT06 produces expected ballooning-like turbulence in full DIII-D X-point geometry

Consider the following simplified equation set in the BOUT06 framework:

$$\frac{\partial N_{i}}{\partial t} + (V_{E} + V_{\parallel}) \bullet \nabla N_{i} = \left(\frac{2c}{eB}\right) b_{0} \times \kappa \bullet (\nabla P_{e} - N_{i}e\nabla \phi) + \nabla_{\parallel}(j_{\parallel}/e) - N_{i}\nabla_{\parallel}V_{\parallel i}$$

### $\frac{\partial \varpi}{\partial t} + V_E \bullet \nabla \varpi = 2\omega_{ci}b_0 \times \kappa \bullet \nabla P + N_i Z_i e^{\frac{4\pi V_A^2}{c^2}} \nabla_{\parallel} j_{\parallel}$

$$\frac{\partial V_{\parallel e}}{\partial t} = -\frac{e}{m_e} E_{\parallel} - \frac{1}{Nm_e} (T_e \nabla_{\parallel} N_i) + 0.51 v_{ei} j_{\parallel}$$

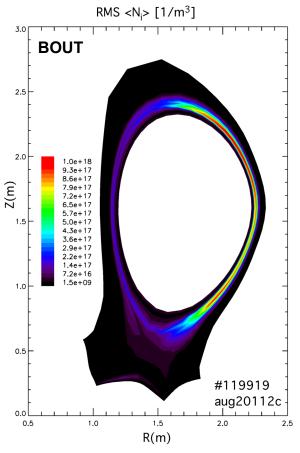
$$\frac{\partial V_{\parallel i}}{\partial t} + V_E \bullet \nabla V_{\parallel i} = -\frac{1}{N_i M_i} \nabla_{\parallel} P$$

$$\mathbf{E} = -\frac{1}{c} \frac{\partial}{\partial t} \mathbf{A}_{\parallel} - \nabla \phi, \quad -\nabla_{\perp}^{2} \mathbf{A}_{\parallel} = \frac{4\pi}{c} \mathbf{j}_{\parallel}, \quad \mathbf{B} = \nabla \times \mathbf{A}_{\parallel} + \mathbf{B}_{0}$$

$$\boldsymbol{\varpi} = \nabla \cdot (eZ_i N_i \nabla \phi) \approx eZ_i N_i \nabla^2 \phi \quad \nabla_{\parallel} \approx \mathbf{b}_0 \cdot \nabla$$

- Electromagnetic with  $\nabla_{\parallel} \approx \mathbf{b}_0 \cdot \nabla$
- Actual DIII-D geometry
- DIII-D like fixed background profiles for shot 119919
- No T<sub>e</sub> fluctuations

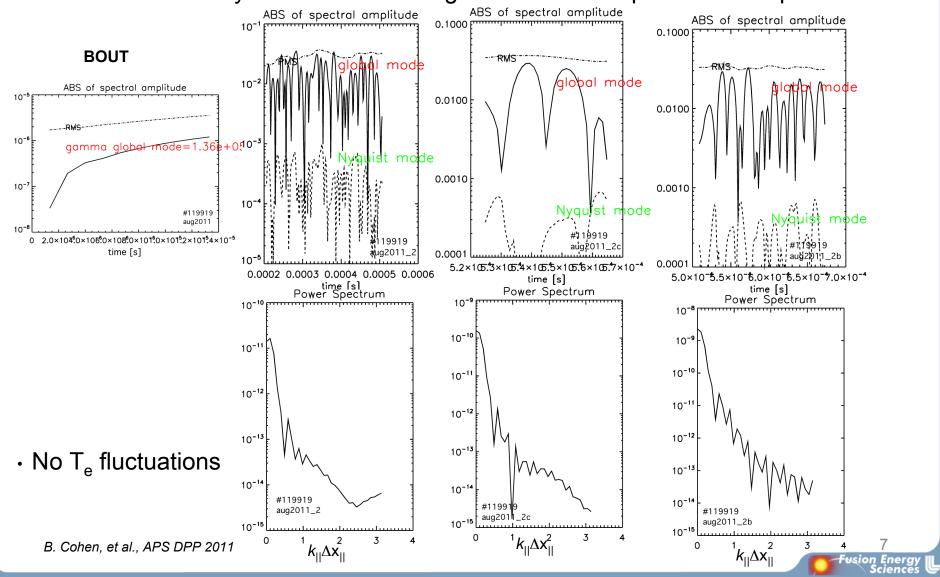
Distribution of  $<\delta N_i>$  in saturated turbulence



B. Cohen, et al., APS DPP 2011

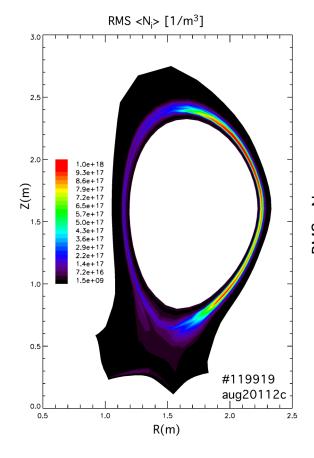
## BOUT-06 produces saturated turbulence for DIII-D geometry with no $T_{\rm e}$ fluctuations

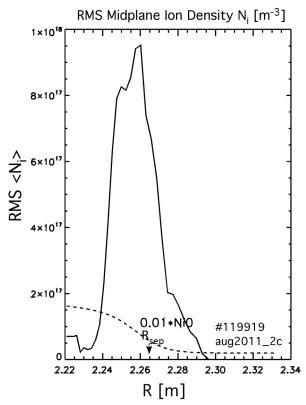
• Evolution of density fluctuations leading to saturated amplitudes and spectra

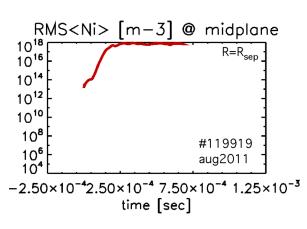


# Time-averaged ion density fluctuations in the midplane saturate at ~10% and peak near $R_{\text{sep}}$

#### **BOUT**



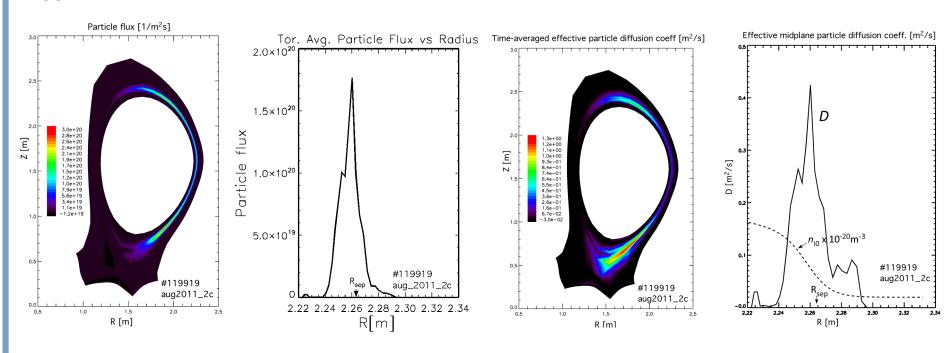




 $\cdot$  No  $\rm T_e$  fluctuations

# Time-averaged ion particle diffusion coefficient saturates at O(0.4) m<sup>2</sup>/s in the midplane and peaks near R<sub>sep</sub>

#### **BOUT**



- No T<sub>e</sub> fluctuations
- In this model with no temperature fluctuations and if  $\nabla \ln(T_{eq}) = \nabla \ln(n_{eq})$ , then  $\chi_{conv} \approx (3/2)D$



## Case #2: Include Advection of Temperature $T_{\rm e}$ in BOUT06 Equations for Resistive Ballooning

Consider the following simplified equation set in the BOUT06 framework:

$$\frac{\partial N_{i}}{\partial t} + (V_{E} + V_{\parallel}) \bullet \nabla N_{i} = \left(\frac{2c}{eB}\right) b_{0} \times \kappa \bullet (\nabla P_{e} - N_{i}e\nabla \phi) + \nabla_{\parallel}(j_{\parallel}/e) - N_{i}\nabla_{\parallel}V_{\parallel i} \quad \text{RMS[eV]}$$

$$\frac{\partial \boldsymbol{\varpi}}{\partial t} + V_E \bullet \nabla \boldsymbol{\varpi} = 2\omega_{ci}b_0 \times \kappa \bullet \nabla P + N_i Z_i e^{\frac{4\pi V_A^2}{c^2}} \nabla_{\parallel} j_{\parallel}$$

$$\frac{\partial V_{\parallel e}}{\partial t} = -\frac{e}{m_e} E_{\parallel} - \frac{1}{Nm_e} (T_e \nabla_{\parallel} N_i) + 0.51 v_{ei} j_{\parallel}$$

$$\frac{\partial V_{\parallel i}}{\partial t} + V_E \bullet \nabla V_{\parallel i} = -\frac{1}{N_i M_i} \nabla_{\parallel} P$$

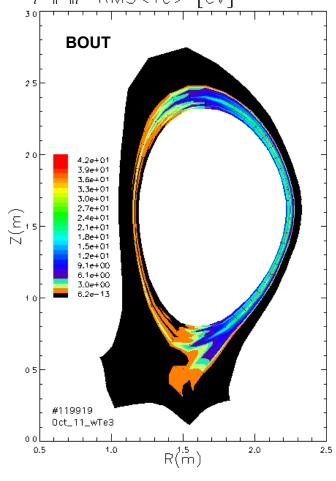
$$\frac{\partial T_e}{\partial t} + V_E \bullet \nabla T_{e0} = 0$$

$$\mathbf{E} = -\frac{1}{c} \frac{\partial}{\partial t} \mathbf{A}_{\parallel} - \nabla \phi, \quad -\nabla_{\perp}^{2} \mathbf{A}_{\parallel} = \frac{4\pi}{c} \mathbf{j}_{\parallel}, \quad \mathbf{B} = \nabla \times \mathbf{A}_{\parallel} + \mathbf{B}_{0}$$

$$\boldsymbol{\varpi} = \nabla \cdot (eZ_i N_i \nabla \phi) \approx eZ_i N_i \nabla^2 \phi \quad \nabla_{\parallel} \approx \mathbf{b}_0 \cdot \nabla$$

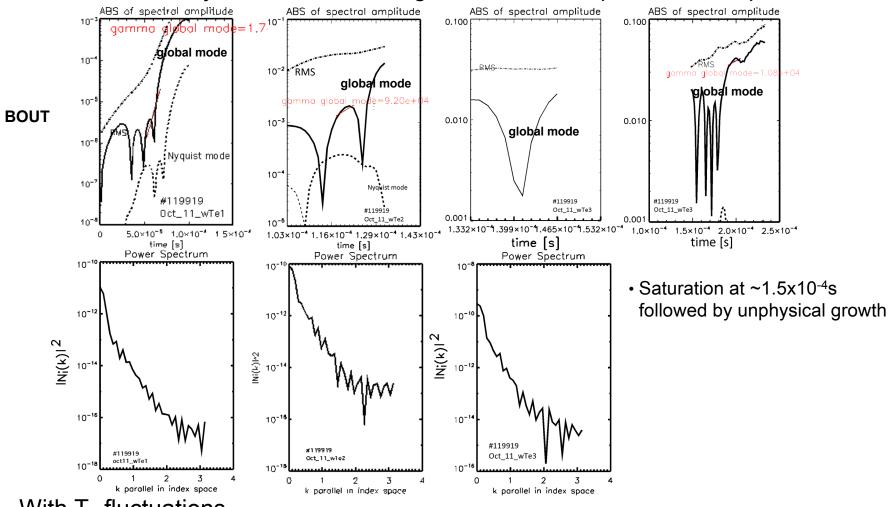
- Electromagnetic with  $\nabla_{\parallel} \approx \mathbf{b}_0 \cdot \nabla$
- Actual DIII-D geometry
- DIII-D like fixed background profiles for shot 119919
- Includes T<sub>e</sub> fluctuations

B. Cohen, et al., APS DPP 2011



## BOUT-06 produces saturated turbulence for DIII-D geometry with T<sub>e</sub> fluctuations

Evolution of density fluctuations leading to saturated amplitudes and spectra



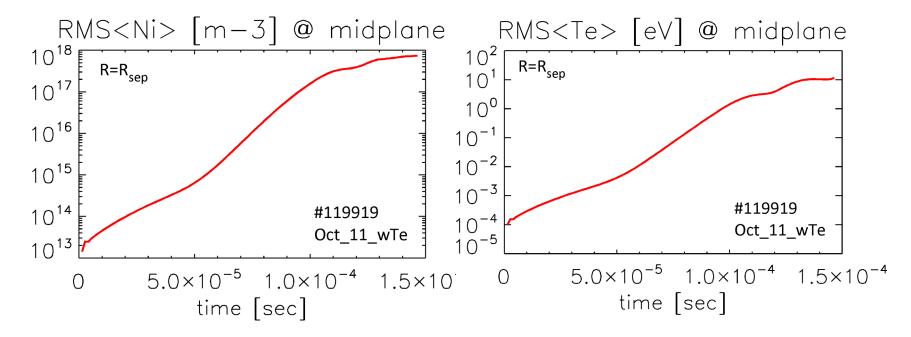
With T<sub>e</sub> fluctuations

B. Cohen, et al., APS DPP 2011



### Ion density and electron $T_e$ fluctuations in the midplane saturate at ~1.5x10<sup>-4</sup> sec

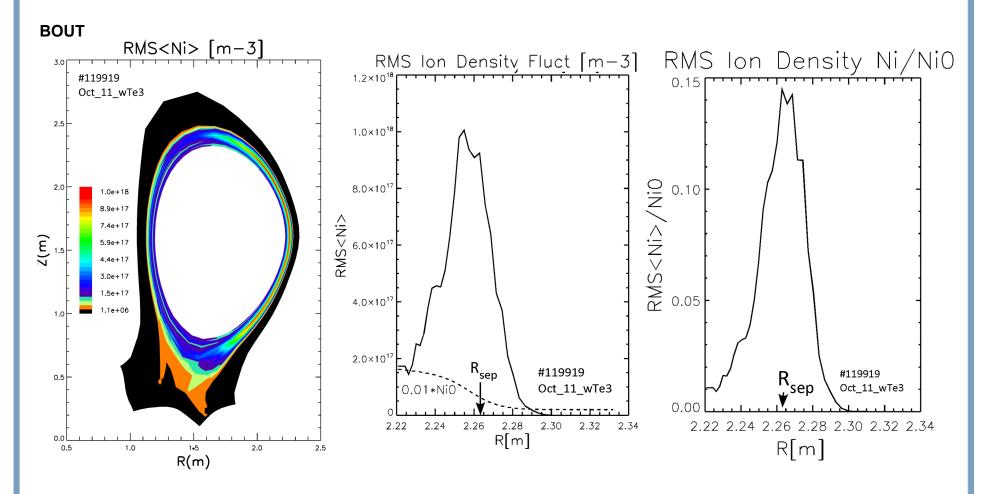
**BOUT** 



With T<sub>e</sub> fluctuations

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# Time-averaged ion density fluctuations in the midplane saturate at ~15% relative amplitude and peak near R<sub>sep</sub>

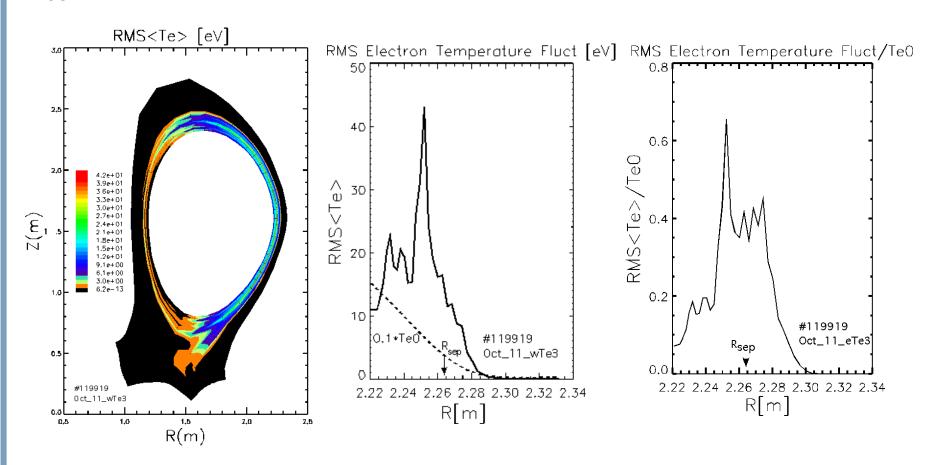


With T<sub>e</sub> fluctuations

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# Time-averaged T<sub>e</sub> fluctuations in the midplane saturate at ~40-60% relative amplitude and peak near R<sub>sep</sub>

#### **BOUT**

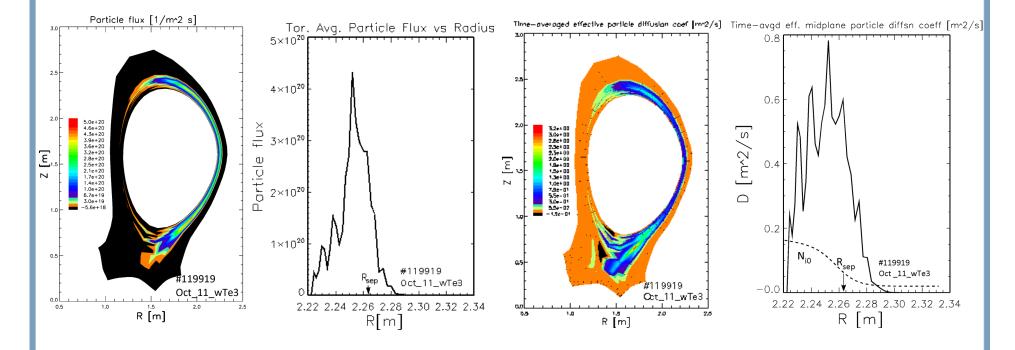


With T<sub>e</sub> fluctuations

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# Time-averaged ion particle diffusion coefficient in the midplane saturates at O(1) m<sup>2</sup>/s and peaks near R<sub>sep</sub>

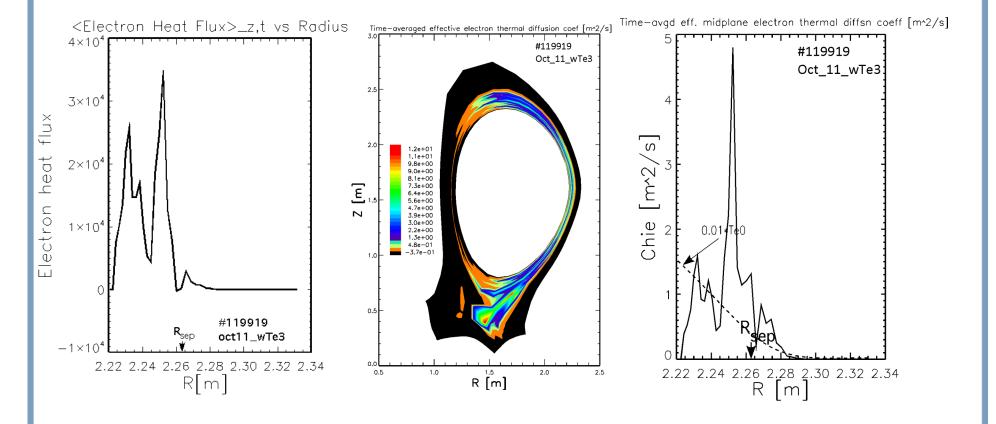
#### **BOUT**



With T<sub>e</sub> fluctuations

# Time-averaged electron thermal diffusion coefficient in the midplane saturates at 1-5 m<sup>2</sup>/s and peaks near R<sub>sep</sub>

### • Bout with T<sub>e</sub> fluctuations



Note: Here heat flux (conductive) =  $N_0 < \delta \tilde{v}_r \delta T_e >_{tort}$ , and  $\chi_e = -N_0 < \delta \tilde{v}_r \delta T_e >_{tort} / N_0 \nabla T_{e0}$ 

### Case #3: Include Advection of T<sub>e</sub> in BOUT06 Equations for Resistive Ballooning with Parallel Electron Thermal Conduction

Consider the following simplified equation set in the BOUT06 framework:

$$\frac{\partial N_{i}}{\partial t} + (V_{E} + V_{\parallel}) \bullet \nabla N_{i} = \left(\frac{2c}{eB}\right) b_{0} \times \kappa \bullet (\nabla P_{e} - N_{i}e\nabla \varphi) + \nabla_{\parallel}(j_{\parallel}/e) - N_{i}\nabla_{\parallel}V_{\parallel i}$$

$$\frac{\partial \boldsymbol{\varpi}}{\partial t} + V_E \bullet \nabla \boldsymbol{\varpi} = 2\omega_{ci}b_0 \times \boldsymbol{\kappa} \bullet \nabla P + N_i Z_i e^{\frac{4\pi V_A^2}{c^2}} \nabla_{\parallel} j_{\parallel}$$

$$\frac{\partial V_{\parallel e}}{\partial t} = -\frac{e}{m_e} E_{\parallel} - \frac{1}{Nm_e} (T_e \nabla_{\parallel} N_i) + 0.51 v_{ei} j_{\parallel}$$

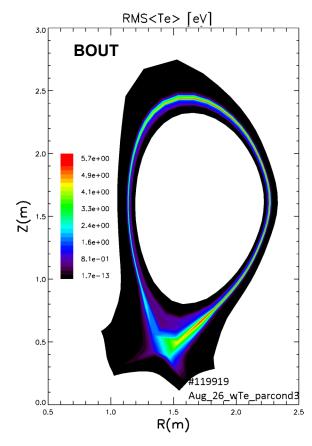
$$\frac{\partial V_{\parallel i}}{\partial t} + V_E \bullet \nabla V_{\parallel i} = -\frac{1}{N_i M_i} \nabla_{\parallel} P$$

$$\frac{\partial T_e}{\partial t} + V_E \bullet \nabla T_{e0} = \frac{2}{3N_0} \nabla \bullet \left( \kappa_{\parallel}^e \nabla_{\parallel} T_e \right), \quad \kappa_{\parallel}^e = 3.2 \frac{N_0 T_{e0} \tau_e}{m_e}$$

$$\mathbf{E} = -\frac{1}{c} \frac{\partial}{\partial t} \mathbf{A}_{\parallel} - \nabla \varphi, \quad -\nabla_{\perp}^{2} \mathbf{A}_{\parallel} = \frac{4\pi}{c} \mathbf{j}_{\parallel}, \quad \mathbf{B} = \nabla \times \mathbf{A}_{\parallel} + \mathbf{B}_{0}$$

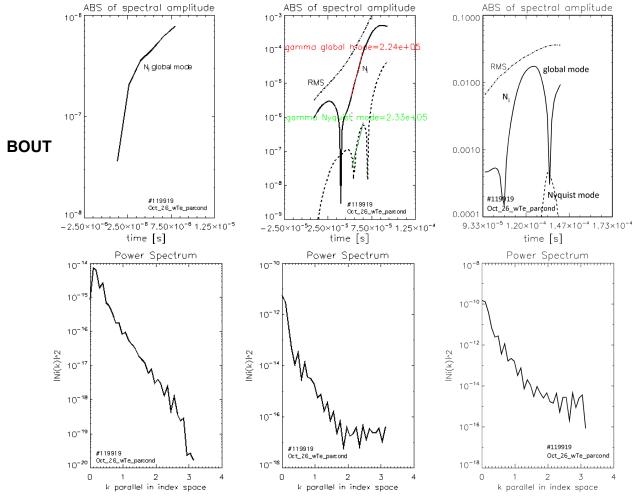
$$\boldsymbol{\varpi} = \nabla \cdot (eZ_i N_i \nabla \varphi) \approx eZ_i N_i \nabla^2 \varphi \qquad \nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla$$

- Electromagnetic with  $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla$
- Actual DIII-D geometry
- DIII-D like fixed background profiles for shot #119919
- Includes T<sub>e</sub> fluctuations & parallel heat conduction



### BOUT-06 produces saturated turbulence for DIII-D geometry with T<sub>e</sub> fluctuations and electron parallel thermal conduction

Evolution of density fluctuations leading to saturated amplitudes and spectra



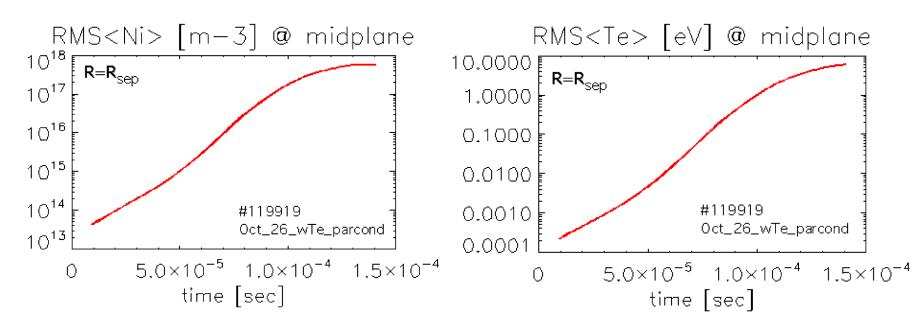
• Saturation at ~1.5x10-4s

• With T<sub>e</sub> fluctuations and electron parallel thermal conduction

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### History of rms fluctuation amplitudes in midplane at separatrix with electron parallel thermal conduction

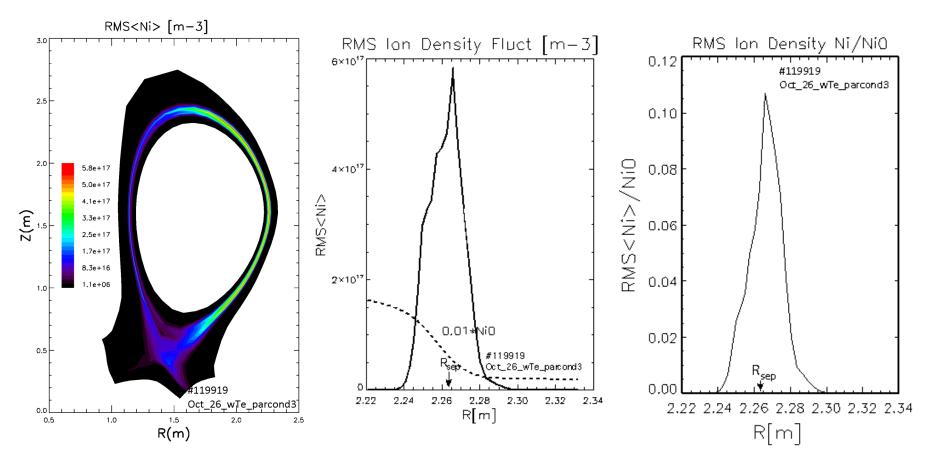
#### **BOUT**



• With T<sub>e</sub> fluctuations and electron parallel thermal conduction

# Time-averaged ion density fluctuations in the midplane saturate at ~11% and peak near $R_{\text{sep}}$

#### **BOUT**

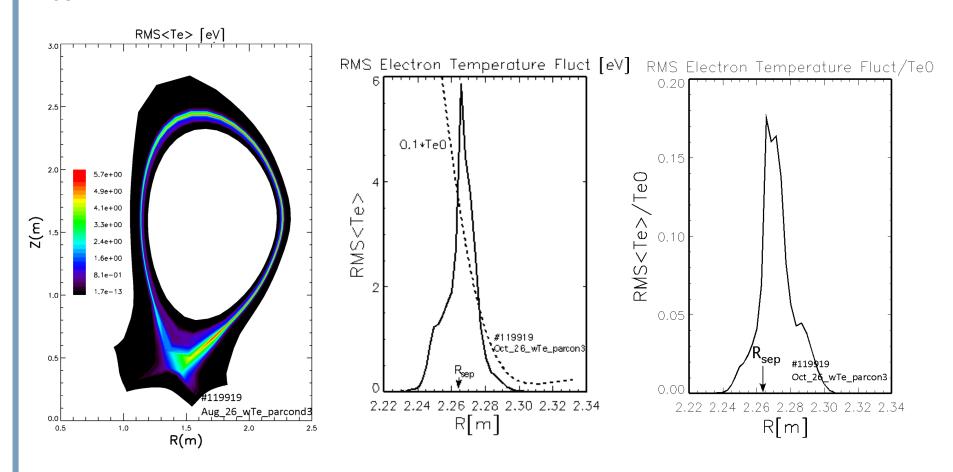


· With T<sub>e</sub> fluctuations and electron parallel thermal conduction



# Time-averaged T<sub>e</sub> fluctuations in the midplane peak near the R<sub>sep</sub> and saturate at ~18% relative amplitude

#### **BOUT**

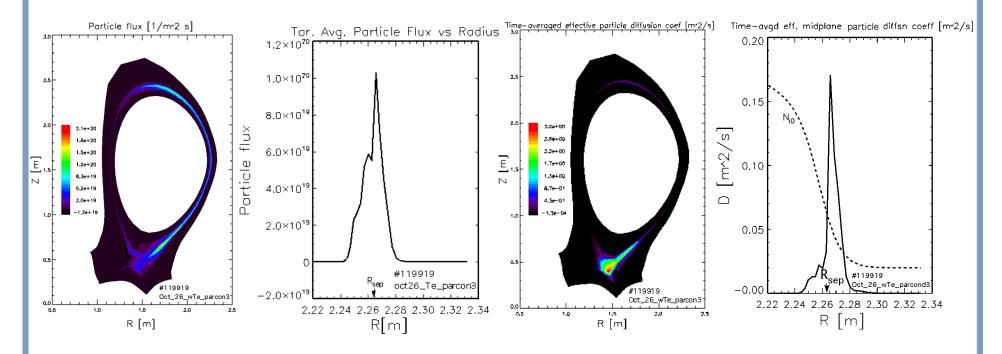


With T<sub>e</sub> fluctuations and electron parallel thermal conduction

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# Time-averaged ion particle diffusion coefficient in the midplane saturates at < 0.2 m $^2$ /s and peaks near R $_{\text{sep}}$

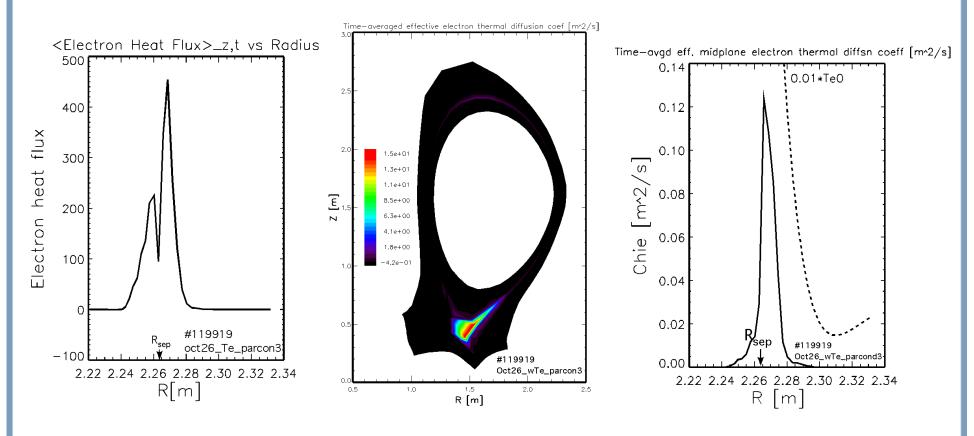
#### **BOUT**



· With T<sub>e</sub> fluctuations and electron parallel thermal conduction

# Time-averaged electron thermal diffusion coefficient in the midplane saturates at ~0.1 m<sup>2</sup>/s and peaks near R<sub>sep</sub>

#### **BOUT**



Note: Here heat flux (conductive) =  $N_0 < \delta \tilde{v}_r \delta T_e >_{tor,t}$ , and  $\chi_e = -N_0 < \delta \tilde{v}_r \delta T_e >_{tor,t} /N_0 \nabla T_{e0}$ 

With T<sub>e</sub> fluctuations and electron parallel thermal conduction

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## Case #4: Include Advection of Temperature $T_{\rm e}$ in BOUT06 Equations for Resistive Ballooning with Magnetic Flutter

Consider the following simplified equation set in the BOUT06 framework:

$$\frac{\partial N_{i}}{\partial t} + (V_{E} + V_{\parallel}) \bullet \nabla N_{i} = \left(\frac{2c}{eB}\right) b_{0} \times \kappa \bullet (\nabla P_{e} - N_{i}e\nabla \varphi) + \nabla_{\parallel}(j_{\parallel}/e) - N_{i}\nabla_{\parallel}V_{\parallel i}$$

$$\frac{\partial \boldsymbol{\varpi}}{\partial t} + V_E \bullet \nabla \boldsymbol{\varpi} = 2\omega_{ci}b_0 \times \kappa \bullet \nabla P + N_i Z_i e^{\frac{4\pi V_A^2}{c^2}} \nabla_{\parallel} j_{\parallel}$$

$$\frac{\partial V_{\parallel e}}{\partial t} = -\frac{e}{m_e} E_{\parallel} - \frac{1}{Nm_e} (T_e \nabla_{\parallel} N_i) + 0.51 v_{ei} j_{\parallel}$$

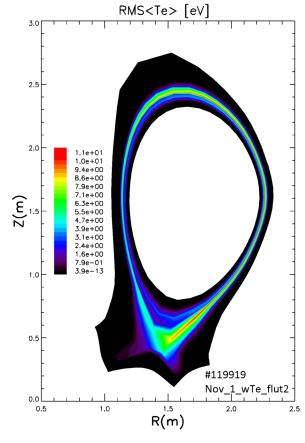
$$\frac{\partial V_{\parallel i}}{\partial t} + V_E \bullet \nabla V_{\parallel i} = -\frac{1}{N_i M_i} \nabla_{\parallel} P$$

$$\frac{\partial T_e}{\partial t} + V_E \bullet \nabla T_{e0} = \frac{2}{3N_0} \nabla \bullet \left( \kappa_{\parallel}^e \nabla_{\parallel} T_e \right), \quad \kappa_{\parallel}^e = 3.2 \frac{n T_{e0} \tau_e}{m_e}$$

$$\mathbf{E} = -\frac{1}{c} \frac{\partial}{\partial t} \mathbf{A}_{\parallel} - \nabla \varphi, \quad -\nabla_{\perp}^{2} \mathbf{A}_{\parallel} = \frac{4\pi}{c} \mathbf{j}_{\parallel}, \quad \mathbf{B} = \nabla \times \mathbf{A}_{\parallel} + \mathbf{B}_{0}$$

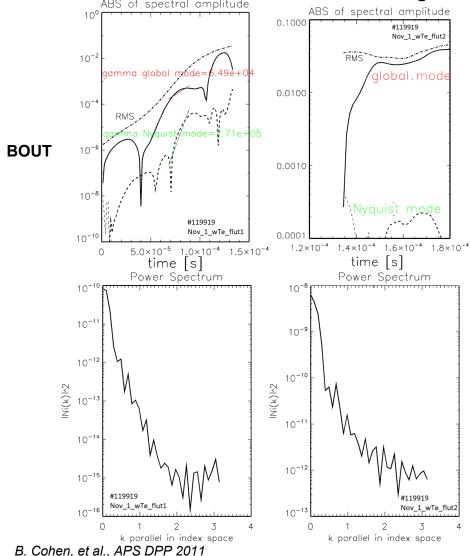
$$\boldsymbol{\varpi} = \nabla \cdot (eZ_i N_i \nabla \varphi) \approx eZ_i N_i \nabla^2 \varphi \qquad \nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$$

- Electromagnetic with  $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$  in the vorticity eqn.
- Actual DIII-D geometry
- DIII-D like fixed background profiles for shot 119919
- Includes T<sub>e</sub> fluctuations and parallel heat conduction



### BOUT-06 produces saturated turbulence for DIII-D geometry with $\delta T_e$ , parallel thermal conduction, and magnetic flutter

• Evolution of density fluctuations leading to saturated amplitudes and spectra



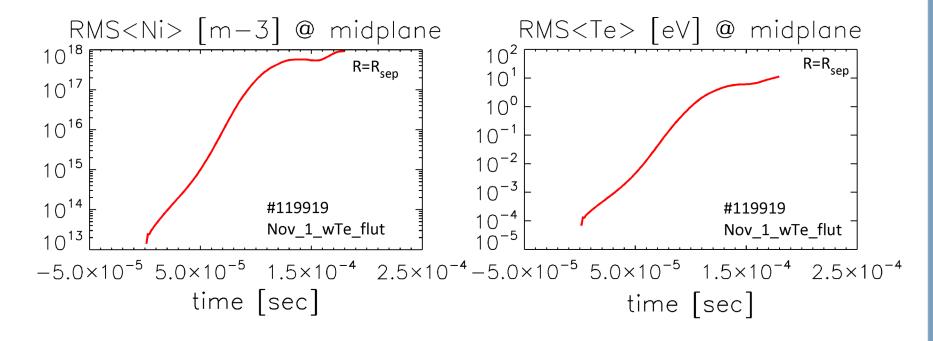
• Saturation at ~1.5x10<sup>-4</sup>s

• With T<sub>e</sub> fluctuations, electron parallel thermal conduction, and

$$\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$$

### History of rms fluctuation amplitudes in midplane at separatrix with electron parallel thermal conduction and magnetic flutter

#### **BOUT**

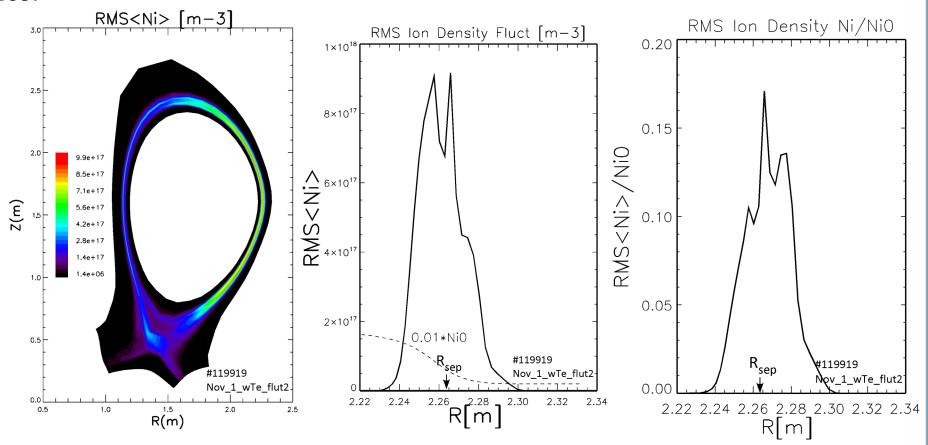


• With  $T_e$  fluctuations, electron parallel thermal conduction, and  $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$  in vorticity equation



# Time-averaged ion density fluctuations in the midplane saturate at ~10-15% and peak near R<sub>sep</sub>

#### **BOUT**

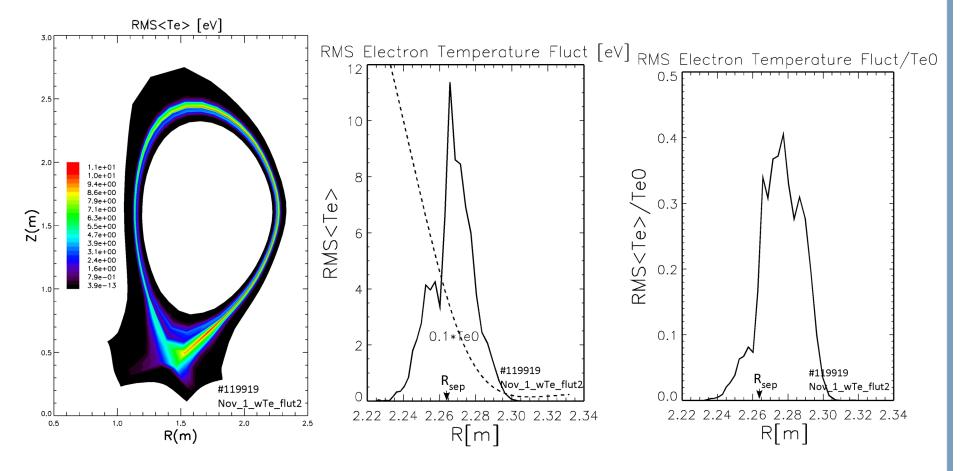


• With  $T_e$  fluctuations, electron parallel thermal conduction, and  $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$ 

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# Time-averaged $T_e$ fluctuations in the midplane peak near the $R_{\text{sep}}$ and saturate at ~25-40% relative amplitude

#### **BOUT**

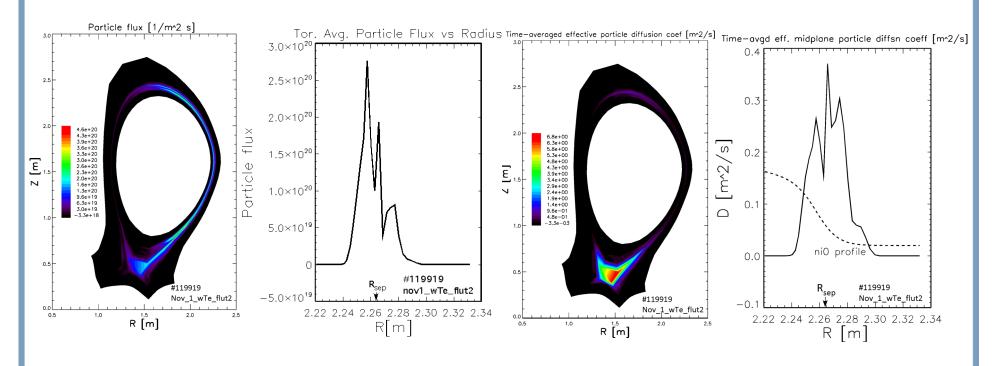


• With  $T_e$  fluctuations, electron parallel thermal conduction, and  $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$ 

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# Time-averaged ion particle diffusion coefficient in the midplane saturates at < 0.3-0.4 m<sup>2</sup>/s

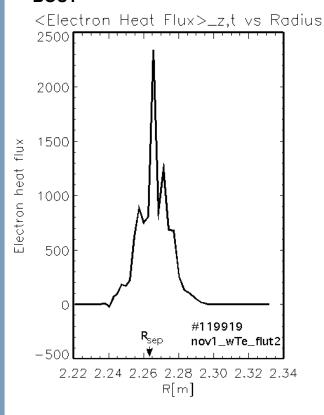
#### **BOUT**

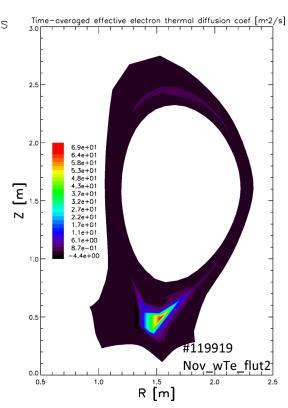


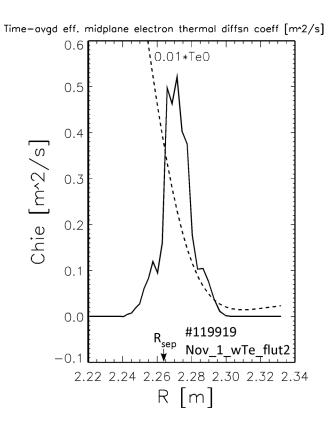
• With  $T_e$  fluctuations, electron parallel thermal conduction, and  $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$ 

# Time-averaged electron thermal diffusion coefficient in the midplane saturates at ~0.5 m<sup>2</sup>/s

#### **BOUT**







Note: Here heat flux (conductive) =  $N_0 < \delta \tilde{v}_r \delta T_e >_{tor,t}$ , and  $\chi_e = -N_0 < \delta \tilde{v}_r \delta T_e >_{tor,t} /N_0 \nabla T_{e0}$ 

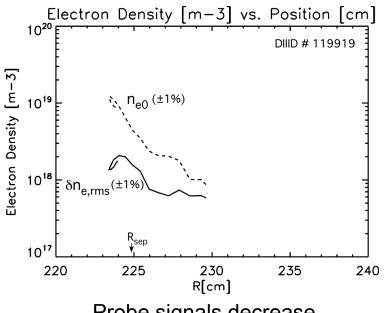
• With  $T_e$  fluctuations, electron parallel thermal conduction, and  $\nabla_{\parallel} = \mathbf{b}_0 \cdot \nabla + \tilde{\mathbf{b}} \cdot \nabla$ 

B. Cohen, et al., APS DPP 2011

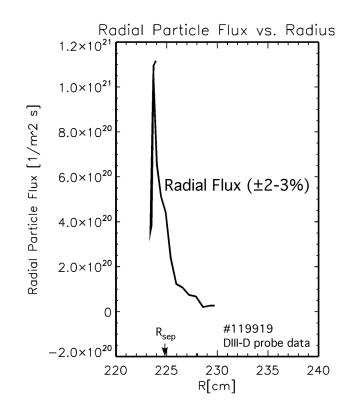


### Langmuir Probe Data for DIII-D #119919 (J. Boedo)

Electron density and radial particle flux vs. radius -- relative density fluctuations exceed ~20%



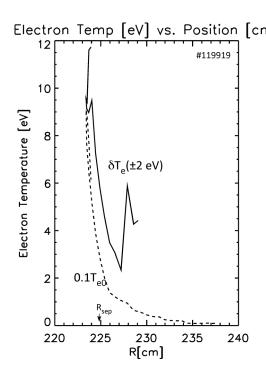
Probe signals decrease below noise levels for R > 229 cm.

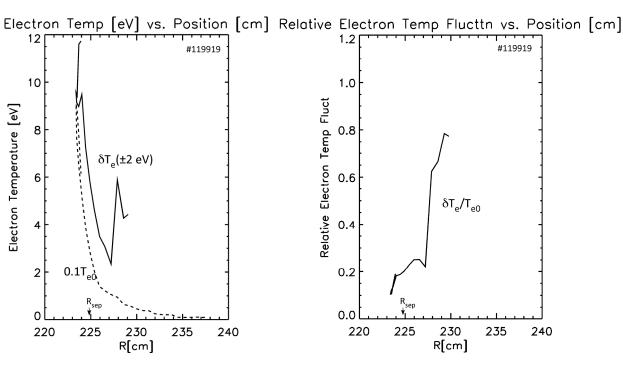


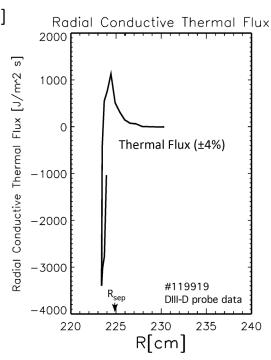
Typical experimental rms density fluctuations at the separatrix are 24-50% There is evidence that  $\delta n$  and the radial flux in the midplane peak near R<sub>sep</sub> as in BOUT results.

### Langmuir Probe Data for DIII-D #119919 (J. Boedo)

### Electron temperature fluctuations in midplane exceed 10%







Probe signals decrease below noise levels for R > 229 cm.

Typical experimental rms  $\delta T_{\rm e}$  fluctuations at the separatrix are 10-25%  $\delta T_{\rm e}$  and the probe fluxes in the midplane usually peak near the separatrix as in BOUT results.

# Summary: As the physics model becomes more complete, the agreement of BOUT results with DIII-D probe data improves

- BOUT algorithmic issues -- control of an odd-even numerical contamination allows us to perform DIII-D simulations
- Comparison of suite of BOUT simulations to shot #119919: peak values in midplane at saturation near R<sub>sep</sub>

· sep						
Bout simulation	$<\delta N_i>_{rms}$ (10 <sup>18</sup> m <sup>-3</sup> )	$<\delta T_e>_{rms}$ (eV)	Radial Particle Flux (10 <sup>20</sup> /m <sup>2</sup> s)	D <sub>r</sub> (m²/s) local	Radial Heat Flux $= \frac{3}{2}N_0 < \delta \tilde{\mathbf{v}}_r \delta \tilde{T}_e > \\ (10^3 \text{ J/m}^2 \text{ s})$	$\chi_{e}(m^{2}/s)$ , local (conductive)
#1: δT <sub>e</sub> =0	0.95	N/A	1.8	0.4	N/A	N/A
#2: δT <sub>e</sub> ≠0 κ <sub>  e</sub> =0	1.0	43	4.3	0.77	54	7.2
#3: δT <sub>e</sub> ≠0 κ <sub>  e</sub> ≠0	0.58	5.8	1.0	0.17	0.72	0.2
#4: $\delta T_e \neq 0$ $\kappa_{\parallel e} \neq 0$ & $\tilde{\mathbf{b}} \cdot \nabla$	0.9	11	2.8	0.38	3.6	0.8
DIII-D #119919 probe data	2.0	10	11.0	~0.15-0.2 ‡	1.2	~0.4 ‡

‡Typical, flux-surface-averaged values for L-mode discharges in DIII-D inferred from UEDGE

B. Cohen, et al., APS DPP 2011

